Mathematical Models Using Isomorphisms and Quantum Formalism, for the Obtention of Risk Wave Functions on Spacecrafts: Application to Voyager-2 Mission of NASA, for Jupiter-Saturn Trajectory. J. C. Echaurren, Codelco Chile Chuquicamata – North Division, jecha001@codelco.cl.

Introduction: The aim this work is to show the results obtained in relation to mathematical models, applied to the risks analysis in spatial activities. The models were designed and applied specifically for the risks control associated to the Voyager program of NASA, being of principal interest the Voyager-2 mission for the encounter with Jupiter and Saturn (July 9, 1979 and August 25, 1981 respectively). Will be specified here the basic concepts used in the development of the models, describing the mathematical and physical formalisms involved, as well as the obtention of orbital-gravitational factors, that will determine the degrees of influence in the manifestation of structural damage, caused by spatial perturbation across of the Jupiter-Sun-Saturn chain, and showing numerical results.

Analytical Method and Results: The model in its fundamental component is based in the Wigner’s Distribution for negative probabilities [2], which is expressed as:

$$ W(x,p) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int x^* xR(x - s/2) \psi xR(x + s/2) \times $$

$$ \times \exp(-ispR) \, ds, $$

$$ = \frac{1}{(2\pi)^{\frac{n}{2}}} \int p^* pR(p + s/2) \psi pR(p - s/2) \times $$

$$ \times \exp(-isxR) \, ds, $$

where xR and pR represent position and momentum respectively, for a vector-risk R in a work activity in static or dynamic regime. The mathematical structure associated to the existence and materialization of risks in accidents, coincides with the abstract form of the isomorphism, where the vector space in the domain is represented by a mathematical subspace \( B \), whose elements are vectors-risk with aleatory behavior; and being the vector space in the codomain, one physical environment, whose elements are quantum probability densities, images of the vector space before mentioned (codomain). These images that can coexist in static and dynamical conditions, are connected by a bijective function \( F \), being the dimension for both vector spaces equivalent to \( n = 4 \), then graphically the functions of link are showed as:

$$ F : B \subset \xi^4 \longrightarrow \xi^4 \equiv F : ( r_1, r_2, r_3, r_4 ) \longrightarrow ( D\psi_1, D\psi_2, D\psi_3, D\psi_4 ). $$

Being the \( r_1 \) and \( D\psi_i \), the mathematical components of a vector R in the vector subspace \( B \), and the components of probability density associated to a wave function \( \psi R \) in the incidents space \( \xi^4 \) respectively, being besides \( r_4 \) and \( D\psi_t \) temporary components in both spaces, where the term \( D\psi_t \) adopts spatial positions of escape, i.e., with a extrapolation out of \( \xi^4 \). The components of the vector space in the codomain generate complete wave functions \( \Psi_i \) associated to the \( R_i \), which possess probabilistic structure or natural, and that satisfy besides the Heisenberg’s Uncertainty Principle [3], therefore: “The probability of to find the risk \( R_i \) defined by the wave function \( \psi R_i( t ) \) in the interval \( dt \) around \( t \) is \( |\psi R_i( t )|^2 dt \)” [1,3]. The complete wave functions \( \Psi_i \) in the vector space \( \xi^4 \) acquire gaussian form, which are pure or perturbed for closed or open systems respectively. Is possible to identify besides the existence of “entropy spaces” \( s \), between \( \xi^4 \) and \( \xi^4 \) that define an unstable regime and aleatory behavior, which are generated in static mode of action, and that are “relative” to the elements of an activity in dynamical mode. Inside of this structure an “incidents space” is defined by, \( E_{inc} \equiv (\xi^4_v) \cup (\xi^4_f) \), being the “accident” defined by \( (\xi^4_f) \cap s \), in dynamical mode and that define a manifestation of energy from \( \xi^4_v \) to \( \xi^4_f \). Then, the entropy space \( s \), is defined by any \( R_i \) that is not included in the procedures of some activity. The formalism before described is applied to activities in the space, working with planetary wave functions, modified as:

$$ \Psi_{\text{spatial}} \equiv (Ags / Agp) (Vvo / Vvop) \Psi_{\text{planet}}, $$

being \( (Ags / Agp) < 1 \) a gravitational factor, and \( 1 < (Vvo / Vvop) < 1 \) an orbital factor, and where:

- \( Ags \) = acceleration of gravity at the space (m/s²).
- \( Agp \) = acceleration of gravity at a planet (p) (m/s²).
- \( Vvo \) = orbital velocity at the space (m/s).
- \( Vvop \) = orbital velocity of a planet (p) around the Sun (m/s).
The orbital factor is approximately obtained through the calculation of escape velocities from both Jupiter and Saturn:

\[
VE = VE_0 + \left( \frac{2\pi R_p}{T_p} \right) + \left( \frac{GM_s}{D_{p,s}} \right)^{1/2} + \left( \frac{2\pi R_p}{8V_0} \right) + \frac{1}{8} \int_0^1 \left( \frac{GM_p}{D^2v,p} \right) dt,
\]

being these the components of: escape velocity from a planet (p), spin velocity for a planet (p), orbital velocity for a planet (p), and gravitational influence associated to a planet (p). These components will influence on both velocity and trajectory of a spacecraft in the space, being besides:

\[R_p = \text{radius of a planet (p)}.
\]
\[T_p = \text{period of a planet (p)}.
\]
\[M_s = \text{mass of the Sun}.
\]
\[D_{p,s} = \text{distance between a planet (p) and the Sun}.
\]
\[M_p = \text{mass of a planet (p)}.
\]
\[Dv,p = \text{distance of approximation from Voyager-2 to planet (p)}.
\]
\[VE_0 = \text{previous escape velocity}.
\]

**Results and Conclusions:** The results obtained are:

a. \((\frac{Ag_s}{Ag \text{ Jupiter }})(\frac{Vv_0 \text{ Voyager-2}}{Vv_0 \text{ Jupiter}}) = 3.18024 = (1.0123)\pi > 1.\)

b. \((\frac{Ag_s}{Ag \text{ Sun }})(\frac{Vv_0 \text{ Voyager-2}}{Vv_0 \text{ Sun }}) = 6.24962E(-8) < 1.\)

c. \((\frac{Ag_s}{Ag \text{ Saturn }})(\frac{Vv_0 \text{ Voyager-2}}{Vv_0 \text{ Saturn }}) = 6.13963 = (1.9543)\pi > 1.\)

The numerical results show an interesting aspect, the involved risks in the Voyager-2 mission, specifically in (a) and (c), reveal an apparent duplication of intensity from Jupiter to Saturn, and this intensity tends to be annulled in the intermediate trajectory, i.e., between Jupiter and Saturn, by a reduced influence from the Sun. The numerical results show major probability of fall in Saturn that in Jupiter. According to this, it is possible to deduce initially:

1. The intensity of the risks varies inversely with the mass of the planets.
2. A great proximity is observed with both values of \(\pi\) and factors of \(\pi\), which indicates a numerical and mathematical structure.
3. The numerical variations are determinable in the time.

These numerical models are feasible of application to a great variety of spatial activities, including both new Exploration Programs and new Mars Exploration Rovers.

**References:**