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A TEST OF SELF-STATIONARITY

By

Robert D. Regan

July 1971

Prepared under NASA Contract H-82013A

This report is preliminary and has not been edited or reviewed for conformity with U.S. Geological Survey standards and nomenclature.

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The work presented in this report was accomplished as part of a study investigating the analysis of traverse geophysics data. The study was directed toward developing methods of extracting the maximum amount of information from geophysical measurements obtained during proposed automated lunar vehicle traverses.

However, it became apparent that a computer program for testing stationarity would be of use in many fields of investigation. Thus it was decided to publish this report separately.
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A TEST OF SELF-STATIONARITY

by

Robert D. Regan

ABSTRACT

A method for testing the stationarity of a single time series has been devised. The method utilizes two pre-existing tests of stationarity. A computer program that permits routine testing of any time series has been developed, and the test has been successfully applied to several known series. In certain cases the self-stationarity determined can be equated to stationarity.

INTRODUCTION

In time-series analysis of any body of observational data, the fundamental assumption is that of stationarity. Very little work has been done with nonstationary time series, and the effects of nonstationarity upon time-series analysis are little known. In the practical application of time-series techniques the assumption of stationarity is routinely invoked without adequate tests for the validity of such an assumption. Yet the utilization of some of the more essential techniques of time-series analysis, such as the Wiener-Khintchine theorem in the indirect method of power spectra computation, is valid only for stationary series.

This situation stems from the fact that, to date, no rapid, accurate, routine method has been available to test the stationarity of a time series. Such a test is clearly required so that stationarity or nonstationarity of a series can be routinely determined and the validity of the application of time-series analysis be established. The method and computer program outlined in this report is designed to fulfill this requirement.

Basically, the method outlined and programmed is a combination of tests proposed by Bendat and Piersol (1966) and Bryan (1967). Their tests and assumptions have been combined with some additional logic to produce a computer test of the stationarity of a single time series. The method has proved successful in several test cases.
BRIEF THEORETICAL SUMMARY

Before the test method and rationale are explained it is essential to discuss some of the concepts and terms pertinent to stationarity.

Time Series-Stochastic Process

A stochastic process can be considered as being composed of a family of random variables, and viewed as a function of two variables \( t \) and \( w \). The sample space associated with this stochastic process is doubly infinite and the set of time functions which can be defined on this space is called an ensemble.

A strict definition of a time series is that it is one realization (outcome) of a stochastic (random) process. A more popular definition is that it is a set of observations of a parameter arranged sequentially.

The label "time-series" is perhaps a misnomer, and is directly applicable only when the observations are made chronologically. However, for the independent variable, time, there may be substituted any other parameter - e.g., distance.

The basic idea of the statistical theory of time series analysis is to regard the time series as a set of observations made on a family of random variables, i.e., for each \( t \) in \( T \), \( x(t) \) is an observed value of a random variable. The set of observations \( \{ x(t); t \in T \} \) is called a time series.

Stochastic Process Functions

The basic process functions are

\( a) \) mean of the process

\[
\mu_X(t) \triangleq E\{x(t)\} = \int_{-\infty}^{\infty} x f(x,t) \, dx
\]

where:

\( \int \) = Lebesgue Integral

\( f(x,t) \) = Probability density function

\( E \) = Expectation operator
In the general case the means are different at different times and must be calculated for each $t$.

b) autocorrelation function of the process

$$R_x(t_1, t_2) = \mathbb{E} \{ x(t_1), x(t_2^*) \}$$

where:

$x(t_2^*) = \text{complex conjugate of } x(t_2)$

c) autocovariance function of the process

$$C_x(t_1, t_2) = \mathbb{E} \{ [x(t_1) - \mu_x(t_1)], [x(t_2) - \mu_x(t_2)] \}$$

In the general case these functions must be calculated for each $t_1$, $t_2$ combination.

Stationarity

In general the properties of a stochastic process will be time dependent. A simplifying assumption which is often made is that the series has reached some form of steady state in the sense that the statistical properties of the series are independent of absolute time. Stationarity can be pictured as the absence of any time-varying change in the ensemble of member functions as a whole.

A sufficient degree of stationarity for most time-series analysis is wide-sense stationarity. A process has wide-sense stationarity if its expected value is a constant and its autocorrelation depends only on the time difference, $t_1 - t_2 = \tau$, and not on the absolute value of the respective times.

i.e.

$$\mathbb{E} \{ x(t) \} = \mu_x(t) = \text{constant}$$

and

$$\mathbb{E} \{ x(t + \tau), x(t)^* \} = R_x(\tau)$$

Wide-sense stationarity is also termed stationarity of the second order, i.e., the series is stationary through its second order statistical moments. Stationarity of order $n$ implies that all statistical moments less than $n$ depend only on the time differences.
Ergodicity

Ergodicity relates to the problem of determining the statistics of the stochastic process from the statistics of one time series. A stochastic process for which the statistics are thus determinable is said to be ergodic, and the single time series is representative of the ensemble. The ensemble moments can then be equated to the time moments.

For example the time average equals the ensemble average of an ergodic process

\[ \mu_x(t) = \frac{1}{T} \int_{-T}^{T} x(t) \, dt = E[x(t)] \]

If a time series is ergodic we need only to measure the time averages which are available rather than the postulated ensemble averages. Since ergodicity is a subclass of stationarity, a time series must be shown to be stationary before the question of ergodicity can be considered.

STATIONARITY OF A SINGLE TIME SERIES

In the transformation from the theoretical to the empirical, strict adherence to theory must be tempered with reasonable practical considerations. In most practical applications the single observed time series is the only information available on the parent process. Hence ergodicity must be assumed and time-domain statistics utilized. Also all analyses are performed on the sample series and thus the stationarity of this series is of prime interest. Bendat and Piersol (1966) have termed the concept of stationarity of this one time series as self-stationarity. Thus we speak of the series being stationary rather than the process being stationary.

However, the concept of self-stationarity is not restrictive. A necessary condition to extend this self-stationarity to stationarity is that the process be ergodic, i.e. that the series is representative of the process. Ergodicity is impossible to prove (except in special instances) when the entire process is not known.
Bendat and Piersol (1966, p. 12) state that "in actual practice, random data representing stationary physical phenomena are generally ergodic".

If the assumption of ergodicity is justified then self stationarity becomes equivalent to stationarity, since the single time series is representative of the ensemble.

**TEST FOR SELF-STATIONALITY**

The test for self-stationarity is based on the methods proposed by Bendat and Piersol (1966) and Bryan (1967). The basis of these methods is that in a stationary series certain statistical properties of the time series are considered invariant with time. The tests are for second order self-stationarity.

**Bendat and Piersol Test**

Bendat and Piersol (1966) suggest that the series be divided into n equal time intervals (either contiguous or non-contiguous) and that the mean and variance of these intervals be calculated. The two series thus formed, composed of means and variances, are then tested for underlying trends or variations by the Run test and the Trend test, Bendat and Piersol (1966, p. 156). If no trends or variations are suggested by the application of these tests, the original series is assumed to be stationary.

**Bryan Test**

Basically, Bryan's test (1967) is quite similar in that he tests the invariance of the means and variances obtained from independent, equal time segments. Rather, than using the sample mean and sample variance he constructs two combinations of the data to serve as estimates of the population mean and population variance. Those two estimates are independent, and independently distributed.

Using these two variates, $m$, a linear function of the data and an unbiased estimate of the population mean, and $Q$, a quadratic function of the data, he develops a test for the hypothesis that the time series is stationary, and two test variables, $L_1$ for
the Neyman-Pearson L test, and F for the F-distribution test.

Test modifications

The tables for the Run and Trend tests (Bendat and Piersol 1966, p. 170) were extrapolated to include the range \( n = 1 \) to \( n = 200 \). With values of \( n \) less than 12 the results proved unreliable. Hence a low limit cut off at \( n = 12 \) is utilized. The Acceptance region was extended to include the lower bound.

The Bryan test was extended to include the 97.5 percent confidence interval.

Self-Stationarity Test

In the proposed test both methods are combined and used with some restrictions and modifications.

First the sampling interval for independent samples is determined. If independent samples cannot be determined, i.e., the autocorrelation function does not damp to zero, the test is aborted and the series can be considered non-stationary if a reasonable number of points has been used.

Once the sampling interval has been determined, the series is segmented into independent samples of length \( N \). Initially the series is tested with \( N = 5 \), then \( N \) is increased to 10, and the final test, if there are enough data points, is for \( N = 15 \).

The minimum test is for \( KK = N \) samples of length \( N \). If there are not enough data points for this number of samples, the requirement for independent samples is relaxed slightly (i.e. the sample separation interval is steadily decreased to a limiting value equal to the number of lags necessary for the autocorrelation function to damp to .100). If at this sample interval there are not \( N \) samples, the test is aborted. If this happens at \( N = 5 \), it may be an indication of nonstationarity.

If we have \( KK \) independent samples of length \( N \) the series is tested for stationarity at three confidence intervals (95%, 97.5%, 99%) in the following manner

a) if \( KK \) is greater than or equal to 12, the Bendat
and Piersol test statistics and the Bryan test statistics are both utilized.

b) if KK is less than 12 only the Bryan test statistics are utilized.

The series is considered stationary if both the Run and Trend tests show no trends or variations for the mean and variance series and if the two test statistics in the Bryan test indicate stationarity.

In the case where only one method is utilized (i.e. KK < 12) stationarity is tested on the merits of the Bryan test statistics alone.

It should be noted that the 95 percent confidence interval is the most restrictive (i.e. the smallest acceptance region) and the other confidence intervals progressively less restrictive.

Also results at the largest N used are preferable since more data points are used in each sample to determine the test statistic and the assumption of normality in the Bryan test statistics is more closely approximated.

COMPUTER PROGRAM STEST

The stationarity test has been programmed on an IBM 360/30 as computer program STEST. A copy of the computer program is contained in the Appendix.

The program requires approximately 53,000 bytes of storage and to test a series of 400 data points for all values of N requires approximately 2 minutes of computer time.

Program Input

The input to the program is simply the number of data points in the series and the values of the data points. An example is contained in the Appendix.

Program Output

The program output has several forms. Initially the lag required for the autocorrelation function to damp to zero is indicated along with the value of the autocorrelation function at
that lag.

If the autocorrelation function does not damp to zero, a statement is printed stating that this occurred and that it may be indicative of non-stationarity.

The stationarity test is then conducted for values of \( N = 5, 10, 15 \). If at any value there are not at least \( N \) samples available for testing, the sampling interval is progressively decreased to a limiting value to obtain \( N \) values. If \( N \) values are obtained in this manner a statement indicating that correlated samples are being used along with the autocorrelation lag and value at this sample interval is printed out. If \( N \) samples cannot be obtained the test is aborted.

If there are \( KK \) samples \( (KK > N) \) of length \( N \), the test results are printed out for the 95\%, 97.5\%, and 99\% confidence intervals.

A sample output is contained in the Appendix.

TEST CASES

Seven series that have been tested are series A-F as given in Box and Jenkins (1970) and a second order auto regressive process as given in Jenkins (1968). The results are shown in Table 1. In all cases except series A the test accurately indicated the stationarity or non-stationarity of the series. The discrepancy in series A may be attributable to the fact that correlated samples were used.

It is interesting to note that the series generated from the second order AR process is stationary and the process itself is stationary. Thus, in this case self-stationarity is indicative of stationarity.
Table 1

Series A - Non stationary

STEST Results
- Correlated samples used -

A) N = 5
95% confidence interval : non stationary
97.5% confidence interval : stationary
99% confidence interval : stationary

B) N = 10
Not enough data points for independent or correlated samples.

Series B - Non stationary

STEST Results

A) N = 5
There are not enough data points for independent or correlated samples. Since this occurred for a sample of length 5 and the length of the input series is 369, this may be indicative of non-stationarity.

Series C - Non stationary

STEST Results

A) N = 5
95% confidence interval : non stationary
97.5% confidence interval : non stationary
99% confidence interval : non stationary

B) N = 10
Not enough data points for independent or correlated samples.
Series D - Non stationary

STEST Results

A) N = 5
For N = 5 there are not enough data points for independent or correlated samples. Since this occurred for a sample of length 5 and the length of the input series is 310 this may be indicative of non-stationarity.

Series E - Stationary

STEST Results

A) N = 5
95% confidence interval : non stationary
97.5% confidence interval : stationary
99% confidence interval : stationary

B) N = 10
Not enough data points for independent or correlated samples.

Series F - Stationary

STEST Results

A) N = 5
95% confidence interval : stationary
97.5% confidence interval : stationary
99% confidence interval : stationary

B) N = 10
Not enough data points for independent or correlated samples.
## Second Order A. R. Process - Stationary

### STEST Results

<table>
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<tr>
<th>N</th>
<th>Confidence Interval</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>95%</td>
<td>stationary</td>
</tr>
<tr>
<td></td>
<td>97.5%</td>
<td>stationary</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>stationary</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>N</th>
<th>Confidence Interval</th>
<th>Result</th>
</tr>
</thead>
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<tr>
<td>10</td>
<td>95%</td>
<td>stationary</td>
</tr>
<tr>
<td></td>
<td>97.5%</td>
<td>stationary</td>
</tr>
<tr>
<td></td>
<td>99%</td>
<td>stationary</td>
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<th>N</th>
<th>Confidence Interval</th>
<th>Result</th>
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<tbody>
<tr>
<td>15</td>
<td>95%</td>
<td>stationary</td>
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<td></td>
<td>97.5%</td>
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</tr>
<tr>
<td></td>
<td>99%</td>
<td>stationary</td>
</tr>
</tbody>
</table>
Appendix
Computer Program STEST
THE PROGRAM USES TWO TESTS

1) J. G. Bryan—Statistical Test of the Hypothesis That a
Time Series Is Stationary—Geophysics, Vol. 32, No. 1

2) Bendat and Piersol—Measurement and Analysis of
Random Data—P. 219

THE SERIES IS READ INTO THE PROGRAM, THE ACF IS CALCULATED AND
THE NUMBER OF LAGS NECESSARY FOR THE ACF TO DAMP TO ZERO IS
DETERMINED. THE SERIES IS THEN SAMPLED AT INTERVALS SEPARATED
BY THIS LAG. THIS ASSURES INDEPENDENCE. THE SAMPLED SERIES IS
THEN TESTED FOR STATIONARITY. IF THE NUMBER OF SAMPLES IS 12 OR
Larger, BOTH TESTS ARE USED. IF LT 12 ONLY BRYAN'S TEST IS USED.

INPUT DATA

NN—LENGTH OF THE SERIES
X—TIME SERIES

C PROGRAM TEST
C THIS PROGRAM TESTS A TIME SERIES FOR WIDE SENSE STATIONARITY
C
C THE PROGRAM USES TWO TESTS
C
C 1) J. G. BRYAN—STATISTICAL TEST OF THE HYPOTHESIS THAT A
TIME SERIES IS STATIONARY—GEOPHYSICS—VOL. 32—NO. 1

DIMENSION X(1000), P(300), W(300), WX(300), WWW(15, 15), RW(300), NTEST(4)

READ(1, 100)(X(I), I=1, NN)

READ(1, 101)(X(I), I=1, NN)

CALL XVAR(X, 1, NN, XRAR, VARX)

ITAU=0
NN=NN
M=0.3*NN

DO 13 IP=1, M

DO 10 IN=1, INP

ASUM=0.

DO 14 IN=1, INP

ASUM=ASUM+((X(IN)-XRAR)*(X(IN+IP)-XRAR))

R(IP)=(ASUM/INP)/VARX

IF(ITAU,G.T.0.) GO TO 12

IF(IP.GT.LE.0.) ITAU=IP

P(IP+1)=R(IP)

CONTINUE
IF (ITAU .GT. 0) WRITE (3, 701)
0021 701 FORMAT ('1,4X,'**** LAG INFORMATION ****,//')
0022 IF (ITAU .LE. 0) WRITE (3, 702) N
0023 702 FORMAT ('//,40X,'**** STATIONARITY TEST FOR N = ',13,1X,'++++');/
0024 ICNT = 0
0025 ISEG = N + ITAU
0026 KK = NN / ISEG
0027 IF (KK .LT. N) CALL PFXIRW, ITAU, N, ISEG, KK, NN
0028 IF (ITAU .LE. 0) WRITE (3, 304) N
0029 304 FORMAT ('//,10X,'FOR N = ',13,1X,'++++');/
0030 DT = N / (KK * ISEG)
0031 NNN = J
0032 MNN = J + (N - 1)
0033 DO 40 J = 1, N, DT
0034 ICNT = ICNT + 1
0035 WX (ICNT) = X (IJ)
0036 40 CONTINUE
0037 CONTINUE
0038 IF (KK .LT. 12) GO TO 445
0039 CALL BPTST (KK, N, WX, NTEST)
0040 445 DO 60 J = 1, N
0041 DO 60 I = 1, N
0042 JI = J - 1
0043 WWW (IJ, J) = W ((JI * N) + 1)
0044 CALL BTST (N, WWW, RW, JTEST)
0045 DO 60 I = 1, 3
0046 IF (KK .LT. 12) NTEST (II) = 4
0047 NTEST (II) = NTEST (II) + JFST (II)
0048 GO TO (130, 131, 132), II
0049 130 CONI = 95.0
0050 GO TO 133
0051 131 CONI = 97.5
0052 GO TO 133
0053 132 CONI = 99.0
0054 133 IF (NTEST (II) .GE. 5) WRITE (3, 788) CONI
0069     IF(NNTST(I) .GE. 5) GO TO 1000
0070     WRITE(3,789) ICONI
0071     789 FORMAT(20X,F5.1,1X,'PERCENT CONFIDENCE INTERVAL.',10X,'THE DATA TE
0072             1STS STATIONARY',/)
0073              /)
0074     1000 CONTINUE
0075     20 CONTINUE
0076     999 STOP
0077     END
SUBROUTINE BPTEST(KK,N,WX,NTEST)
DIMENSION WX(1),XM(200),XXAR(200)
DIMENSION ITEST(4),MT(4),NTEST(11)
ICNT=0
NFI=KK+N
DO 40 J=1,NFI,N
ICNT=ICNT+1
NNN=J
MMN=J+(N-1)
CALL XVAR(WX,NNN,MMN,XXAR)
XME(ICNT)=MMN
XXAR(ICNT)=VAR
40 CONTINUE
NNN=1
KJL=ICNT
MMM=NNN
CALL XVAR(XME,NNN,MMN,XXAR)
CALL RTE(XME,NNN,MMN,XXN,ITEST)
DO 32 KK=1,3
MTEST(KK)=ITEST(KK)
32 MTEST(KK)=ITEST(KK)
NNN=1
MMM=KJL
CALL XVAR(XXAR,NNN,MMN,XXN,VAR)
CALL RTE(XXAR,NNN,MMN,XXN,ITEST)
DO 90 IJK=1,3
NTEST(IJK)=MTEST(IJK)+ITEST(IJK)
90 CONTINUE
RETURN
END
SUBROUTINE XVAR(X,NNN,MMM,XMN,VAR)

DIMENSION X(1)

XSUM = 0.0

DO 244 K = NNN, MMM

XSUM = XSUM + X(K)

CONTINUE

AN = MMM - NNN + 1.0

XMN = XSUM / AN

A = 0.0

DO 220 I = NNN, MMM

A = A + (X(I) - XMN)**2

VAR = A / AN

RETURN

FND
SUBROUTINE RTEST(INN, MNN, MNN, XM, ITEST)

DIMENSION ITEST(1)

DIMENSION XX(100,6), XXX(100,6)

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<th>1.205</th>
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```
MNP = MMM
0007
0008
IF(MMM .GT. 100) MMM = 100.
0009
N = MMM - NNN + 1
0010
CNT = 0.0
0011
L = MMM - 1
0012
DO 430 I = NNN + L
0013
DO 431 J = I, L
0014
IF(XX(J, I+1) = 1) CNT = CNT + 1.0
0015
431 CONTINUE
0016
430 CONTINUE
0017
RUN = 1.0
0018
IF(X(NNN) = XM) 420, 403, 402, 402
0019
402 X(NNN) = 4.0
0020
GO TO 404
0021
404 CONTINUE
0022
NJ = NNN + 1
0023
IF(MNP .GT. 200) MNP = 200.
0024
NN = MNP - NNN + 1
0025
DO 433 JJ = NJ, MNP
```
0027          X(IJJ)=X(IJJ)-XM
0028  IF(X(IJJ).LT.0.0) GO TO 418
0029  419 IF(X(IJJ)-1)=3.01900,420
0030  421 RUN=RUN+1.0
0031  420 X(IJJ)=4.0
0032  GO TO 433
0033  GO TO 433
0034  RUN=RUN+1.0
0035  X(IJJ)=3.0
0036  GO TO 433
0037  CONTINUE
0038  'MN=MN/2
0039  GO TO 444
0040  900 WRITE(3,*03)
0041  503 FORMAT(7H ERROR)
0042  444 ACNT=ACNT
0043  IF(N.GT.100) RETURN
0044  449 IJK=1,3
0045  499 ITFST(IJK)=0
0046  KJI=7-IJK
0047  TEST1=XX(IJK)
0048  TEST2=XX(KJI)
0049  TESTA=XXX(IJK)
0050  TESTB=XXX(KJI)
0051  IF(RUN.LE.TEST2.AND.RUN.GE.TEST1)TEST(IJK)=ITEST(IJK)+1
0052  IF(ACNT.LE.TESTB.AND.ACNT.GE.TESTA)ITEST(IJK)=ITEST(IJK)+1
0053  CONTINUE
0054  RETURN
0055  END
SUBROUTINE RTESTIN,X,WW,JTESTI

DIMENSION Q151,QQ15J,AM151,Z112251,Z1112251,Z111151,JTESTI

DIMENSION X(15,15),WW(15,15),Z11(15,15),Y(15,15)

DIMENSION AAM151,JTESTI

DIMENSION TEST313,3I,F313,31

DATA TEST3/.99,.22,.48,.82,.79,.77,.78,.86,.85/,

DATA F3/2.87,3.51,4.43,1.99,3.31,2.61,1.73,1.92,2.16/

C NOW CONSTRUCT THE AUTOCORRELATION MATRIX Z

K=N

DO 10 J=1,N

10 Z(I,J)=WW(I)

DO 20 J=2,K

J1=J-1

20 Z(I,J1)=WW(I-J1)

DO 30 I=J,N

30 Z(I,J)=WW(I-J)

DO 40 J=1,N

40 Z(I,J)=WW(I-J)

DO 50 I=1,N

Z(I,I)=WW(I)

CONTINUE

C CHANGE THE Z MATRIX TO MULTIPLEXED FORM ZZ FOR INVERSION

K=N

DO 60 J=1,K

60 Z(I,J)=Z(I,J)

C NOW CALCULATE INVERSE OF AUTOCORRELATION MATRIX

CALL MAINEXIT(ZZ,ZIII)

C TRANSFER INVERSE MULTIPLEXED ZIII INTO REGULAR MATRIX ZZ

DO 70 J=1,K

70 Z(I,J)=ZIII(J*J+1)

DO 80 J=1,N

80 Z(I,J)=ZI(I,J)

DO 90 I=1,N

90 Z(I,J)=ZI(I,J)

DO 100 I=1,N

GSUM=0.0

100 Z(I,I)=GSUM

DO 110 J=1,K

110 GSUM=GSUM+ZIII(J*I+I)

DO 120 J=1,N

120 GSUM=GSUM

ASUM=0.0

DO 130 I=1,N

130 ASUM=ASUM+G(I)

DO 140 I=1,N

140 G(I)=GSUM

DO 150 I=1,N

150 ASUM=ASUM

DO 160 I=1,N

160 Z(I,I)=G(I)/ASUM
C C
0038 DO 503 J=1,K
0039 DO 503 I=1,N
0040 R(I,J)=2(I,J)-G(I)*W(J)
0041 503 CONTINUE
C C
0042 DO 504 J=1,K
0043 DO 504 I=1,N
0044 YSUM=0.0
0045 DO 505 KK=1,N
0046 505 YSUM=YSUM+B(I, KK)*X(KK,J)
0047 Y(I,J)=YSUM
0048 504 CONTINUE
C C
0049 DO 506 J=1,K
0050 QSUM=0.0
0051 DO 507 I=1,N
0052 507 QSUM=QSUM+X(I, J)*Y(I, J)
0053 QIJ=QSUM
0054 506 CONTINUE
C C
0055 508 CONTINUE
C C
0056 DO 509 I=1,K
0057 IF(QIII.GT.0.)GO TO 900
0058 GO TO 999
0059 900 QQIII=ALOG10(QII)
0060 509 CONTINUE
C C
0061 AK=K
0062 AN=N
0063 QSUM=0.0
0064 QSUM=0.0
0065 DO 510 I=1,K
0066 510 QSUM=QSUM+Q(I)
0067 QSUM=QSUM+QSUM+Q(I)
0068 510 CONTINUE
0069 QMN=QSUM/AK
0070 QOMN=QSUM/AK
C C
0071 FIND ANTILOG OF QQMN
C C
0072 ANOM=10**QQMN
0073 TEST1=ANOM/QMN
0074 QQMN=0.0
0075 QQMN=QQMN/ANOM
0076 QQMN=QQMN/ANOM
C C
0077 SECOND PART OF TEST
C C
C DO 511 J=1,K
C SUMM=0.0
C DO 512 I=1,N
C SUMM=SUMM+X(J,I)*W(I,J)
C AAM(J)=SUMM
C C CALCULATION OF F
C SSSUM=0.0
C SMSUM=0.0
C DO 520 J=1,K
C SMSUM=SMSUM+AAM(J)
C SSSUM=SSSUM+(AAM(J)*AAM(J))
C 520 CONTINUE
C C
C STAR1=(AK1*(SSSUM)-(SMSUM)*(SMSUM))/AK1
C STARM=ASUM*STAR1
C C
C IK1=K-1
C IK2=K*(N-1)
C AK1=IK1
C AK2=IK2
C F1=STAR1/IK1
C F2=ASUM/AK2
C F=F1/F2
C C
C IX=N/5
C DO 1000 IK1=1,3
C JTEST(IK1)=0.0
C TEST2=TEST3(IK1,IX)
C F2=F3(IK1,IX)
C IF(TEST1.GT.TEST2.AND.F.LT.F2)JTEST(IK1)=JTEST(IK1)+1.0
C 1000 CONTINUE
C 999 RETURN
C 993 END
SUBROUTINE PFIXIPW, TAU, N, ISEG, KK, NN

DIMENSION RW(11)

WRITE(3,600) ITAU, N

600 FORMAT(15X,'WITH TAU =',I3,3X,'AND N =',I4,3X,'THERE ARE NOT ENOUGH
14 DATA POINTS FOR INDEPENDENT SAMPLES',/)

ITAU = ITAU + 1

IF(RW(ITAU+1).GT.0.1) ITAU = 0

ISEG = N + ITAU

KK = NN / ISEG

IF(ITAU.LE.0) RETURN

IF(KK.LT.N) GO TO 1

WRITE(3,601) ITAU, ITAU, RW(ITAU+1)

601 FORMAT(25X,'CORRELATED SAMPLES USED WITH TAU =',I3,3X,'AND R(',I3,
1*) =',F7.4,/)...

RETURN

FND
SUBROUTINE MAINE(N,A,B)
C VERSION 1 OF SUBROUTINE MAINE
C SYMMETRIC MATRIX INVERSION BY ESCALATOR METHOD
DIMENSION A(400),B(400)
R(1)=1.0/A(1)
IF(N.EQ.1)RETURN
MM=N*N
DO 5 I=2,NN
5 R(I)=0.0
DO 10 M=2,N
K=M-1
MM=M*(M-1)*N
EK=A(MM)
DO 10 I=1,K
10 DO 15 J=1,K
MI=M+(M-1)*N
IJ=I+J-1*N
JM=J+(M-1)*N
DO 10 EK=EK-A(IMI)*B(IJ)*A(JM)
15 EK=EK-1.0/EK
DO 20 I=1,K
20 IM=I+(M-1)*N
DO 20 J=1,K
20 DO 30 MI=M+(I-1)*N
30 WI=WI+B(IM)*B(IJ)*A(JM)/EK
DO 25 IM=IM+(I-1)*N
30 B(IM)=B(IM)
DO 27 IM=I+(M-1)*N
DO 28 J=1,K
27 DO 28 MJ=M+(J-1)*N
28 B(IJ)=B(IJ)+B(IM)*B(MJ)*EK
DO 29 CONTINUE
25 CONTINUE
RETURN
END
Sample Input
SAMPLE INPUT

(Series C - Box and Jenkins, 1970, p. 528)

0226
26.6
27.0
27.1
27.1
27.1
27.1
27.1
26.9
26.8
26.7
26.4
.
.
.
.
20.2
19.7
19.3
19.1
19.0
18.8
Sample Output
SAMPLE OUTPUT

***** LAG INFORMATION *****

INDEPENDENT TAU = 20 \hspace{1cm} R(20) = -0.0033

***** STATIONARITY TEST FOR N = 5 *****

95.0 PERCENT CONFIDENCE INTERVAL. \hspace{1cm} THE DATA TESTS NONSTATIONARY
97.5 PERCENT CONFIDENCE INTERVAL. \hspace{1cm} THE DATA TESTS NONSTATIONARY
99.0 PERCENT CONFIDENCE INTERVAL. \hspace{1cm} THE DATA TESTS NONSTATIONARY

***** STATIONARITY TEST FOR N = 10 *****

WITH TAU = 20 AND N = 10 \hspace{1cm} THERE ARE NOT ENOUGH DATA POINTS FOR INDEPENDENT SAMPLES

FOR N = 10 \hspace{1cm} THERE ARE NOT ENOUGH DATA POINTS FOR INDEPENDENT OR CORRELATED SAMPLES----TEST ABORTED.
REFERENCES


